A Model of Demographic Cycles in a Traditional Society: The Case of Ancient China

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ABSTRACT

This publication is devoted to the theory of demographic cycles advanced in researches of many authors. F. Braudel named these cycles as 'general cycles', and R. Cameron used concept 'logistics cycles'. The author has constructed the mathematical model of a demographic cycle. The size of sowing areas, population, the number of peasants and handicraftsmen are basic variables of this model. The level of life, reserves of grain, a sale of grounds, a growth of large property, a transition of peasants to tenantry or handicraftsmen and other factors of the socio-economic relations are taken into account. Generally, the model represents an analogue of system of three integro-differential equations.

The verification of the model is made on the material of the history of Ancient China. The good concurrence of the design data to the data of the account of the population is established.

Population growth with limited amount of resources and with a fixed production technology can be described by a 'logistics equation':

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$\frac{dN}{dt} = r\left(1 - \frac{N}{C}\right)N$, where $N(t)$ is population, C is capacity of the ecological niche, and r is rate of natural growth (Pearl 1925). According to this formulation, population grows rapidly at the beginning, but the rate of growth decelerates and approaches C asymptotically (Fig. 1). At the maximum carrying capacity the population consumes the minimum necessary to maintain its numbers. This conceptualization overlooks the fact, however, that overshooting, can actually cause the crude death rate to rise considerably, and can also lead to 'demographic catastrophes', as in Malthus's exposition. After the catastrophe a new demographic cycle can begin.

Recently economic and demographic historians such as F. Braudel (1979), R. Cameron (1989), M. Artzrouni and J. Komlos (1985) have employed such models, mostly for Medieval and early-modern Europe. J. Goldstone (1991) conceives of these as ecological cycles, and P. Turchin (2003) discusses Goldstone's model in his new book.

The purpose of this paper is to construct a compact model describing the basic economic and demographic processes of a pre-industrial society[1]. The verification of model was made by means of data pertaining to the history of China in the 1st and 2nd centuries, as information on population and on sowing areas is extant for this period. These data enable us to compare our simulations to the actual values realized (see: Lee 1921; Chao 1986; Krukov, Perelomov, Sofronov and Cheboksarov 1983; Malijvin 1983).

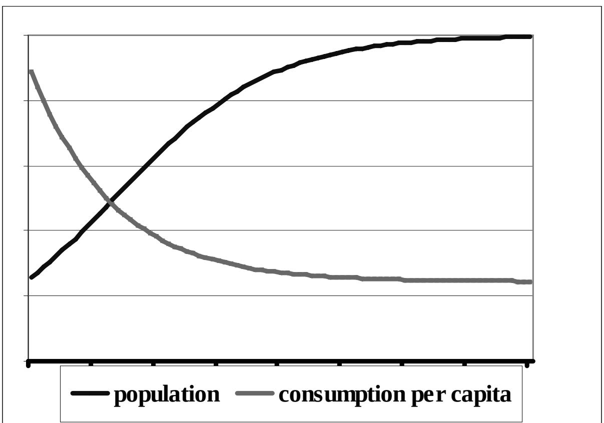

Fig. 1. The logistics curve and the curve of consumption per capita

Let us suppose that agricultural output is characterized by production function:

$$Q(t) = F[P(t)]A \qquad (1)$$

where *P(t) is* the rural population at period *t, A(t)* is a cultivated area and *F(P)* is some function. *Q* is crop output measured in kilograms. Hence, $F(P) = Q/A = k_s p_s$, where p_s is productivity per hectare, and k_s is the multiple-cropping index (sown area divided by cultivated area). The agrarian technology was constant in this traditional society, therefore p_s and k_s do not depend on the population P[2].

Thus, we have the production function $Q = k_s p_s A$. It is necessary to take into consideration that *A* depends on *P*. We use Chinese data to derive this relationship (Table 1).

Table 1

Population and Cultivated Area in China

Year A. D.	Population (million)	Cultivated area (million hectares)
57	21	16.4
88	43.4	33.4
105	53.3	34.1

The graph of cultivated area as a function of the population size is presented in Fig. 2

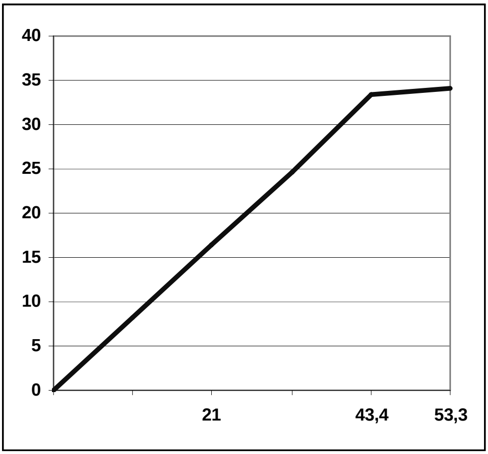

Fig. 2. The relationship between cultivated area and population in Ancient China

The area, A, under cultivation in ancient China obviously increased proportionally to the population P until A reached the maximum value A_m. This relationship can be described by the following function:

$$A(P)= kP, \text{ if } kP < A_m \qquad (2a)$$
$$A(P)= A_m, \text{ if } kP > A_m \qquad (2b)$$

Value A_m was equal to about 34 million hectares in the period under consideration.

The society consisted of peasants, tenants and landlords. Let $Y(t)$ be the number of peasants, and A_F – the area of land belonging to the farmers and A_T, the land of the tenants. We calculate the area of lands belonging to the farmers, A_F, using equations (2a) – (2b) (but the lands of the tenants are deducted from A_m). The land A_T is occupied by the tenants. The maximum land of the peasants is $A_m - A_T$. Then

$$A_F(Y)= kY, \text{ if } kY < A_m - A_T \qquad (2c)$$
$$A_F(Y)= A_m - A_T, \text{ if } kY > A_m - A_T \qquad (2d)$$

Let q be the quantity of seed needed per hectare and let M be the total grain requirements for seed. Then $M = k_s q A_F$. Let p_0 be the minimal consumption per capita; in the case of China, p_0 was equal to about 215 – 230 kg of grain per year.

The value $P_0 = p_0 Y(t)$ is the minimal total consumption, and $W=M+P_0$ is the quantity of grain needed to cover minimum consumption and seed. Let $X(t)$ be the quantity of grain after harvest (crop and stocks). In case $X(t) > W$ the peasants have grain surpluses.

The amount of grain available per capita, u, where

$$u = (X(t) - M)/Y(t).$$

This is not all consumed in the current year, however. Let us assume that half of the surplus is stored for future consumption.

Let p_m designate maximum consumption, then consumption per capita p_c is:

$$p_c = (u + p_0)/2 \text{ if } u > p_0 \text{ and } (u + p_0)/2 < p_m \qquad (3a)$$
$$p_c = p_m \text{ if } (u + p_0)/2 > p_m \qquad (3b)$$

(If $u<p_0$, then $X(t) < W$. This case is considered below). Total consumption is $P_1 = p_c Y(t)$, and total grain output is used for consumption and seed: $W_1=M + P_1$, so by the time of the following crop the available grain stock is $Z_p = X(t) - W_1$. Further, let l_0 be the output of grain per sowing. Certainly, the productivity was not constant, and we take it into account by adding to l_0 the random variable dl_0, so the real productivity becomes $l = l_0 + dl_0$. The production function is $Q = k_s p_s A_F = k_s q l A_F = lM$, then the crop of the next year is equal to lM. It is necessary to subtract the taxes from this quantity. The taxes were equal to 1/30 of a crop and 120 coins from each adult person (23 coins from a teenager). Each person paid 60 coins on average. This computes the grain equivalent of this monetary tax according to market prices, and obtains the total amount of taxes in terms of grain, H.

After the harvest the quantity of grain is equal to

$$X(t+1) = lM - H + X(t) - W_1$$

including the stocks.

Now population $Y(t+1)$ should be determined. In classical model by R. Pearl it is

$$Y(t+1) = \frac{rY(t)}{1+(r-1)\dfrac{Y(t)}{C}} \qquad (4)$$

Here r is the rate of natural growth in favorable conditions, and C is the capacity of an ecological niche or the maximal population at available food resources. In our case $C=P_1/p_0$.

We shall use a more recent model and replace the term $\dfrac{Y(t)}{C}$ by $\left(\dfrac{Y(t)}{C}\right)^n$ where n is a parameter of compensation suggested by J. Maynard Smith and M. Slatkin (1973). The introduction of this parameter is explained by the fact that in human societies famine results not only in high death rate, but also in revolts and wars which increase the death rate even more.

Let us consider now the case $X(t)<W$, when the peasants have grain deficits. Then the peasants lack sufficient grain in spring sowing even if they consume p_0. Then they sell a part of their land in order to compensate for the lack of seed grain. In some cases the landowners have a limited stock of grain and can not buy all the land sold by the peasants, then the peasants reduce their fund of consumption P_1 so, that $M + P_1 = X(t)$. In this case $u < p_0$ and consumption per capita equals $p(u)=P_1/Y(t)$. During the famine $P_1<p_0 Y(t)$ and $Y(t)/C = Y(t)/(P_1/p_0) = p_0 Y(t)/P_1 > 1$ in (4). Therefore population is reduced.

If the famine threatens destruction of a significant part of the population, the authorities distribute grain to the peasants, increasing consumption up to p_{u0} ($p_{u0} < p_0$). As the peasants sell the land, the large landed property gradually grows, and the cultivated area of the peasants decreases. The landowners attract tenants who provide them half of the crop as land rent; hence, a tenant should have twice more ground than a peasant, approximately 1,5 hectares per capita. In case the peasants sold land of area D_a in the current year, it is possible to locate $N_a = D_a/1.5$ tenants on these lands and the peasant population decreases by value N_a.

Let us now consider the dynamic evolution of the number of tenants. Let A_T be an area of grounds of the tenants, $Y_a(t)$ is a number of the tenants in one year t, and $X_a(t)$ are stocks of the grain of the tenants, except for the taxes and sowing fund. Weight of seed grain of the tenants is equal to $M_a = k_s q A_T$, and the minimal total consumption is $P_{a0} = p_0 Y_a(t)$. In case $X_a(t) > P_{a0}$ the tenants have surpluses of grain, and consumption per capita of the tenants (p_{ua}) is calculated just as for the peasants.

The general consumption of the tenants is $P_a = p_{ua} Y_a(t)$, and to the next harvest the stocks $X_a(t) - P_a$ will be saved in barns of the tenants. The crop of the next year is lM_a, and the taxes H_a are calculated just as for the peasants. After a deduction of taxes and seed grain the tenant receives only half of the crop, therefore stocks of grain of the tenants will be equal to $X_a(t+1) = ((l-1)M_a - H_a)/2 + X_a(t) - P_a$.

The maximal number of the tenants is given by $C = P_a/p_0$, and the actual number is determined just as for the peasants. Each year the number of the tenants is increased by number N_a (the number of peasants becoming new tenants). These new tenants receive sites of land about 1,5 hectares, sowing grain and annual norm of consumption. They should return this grain in the future. During famine the landowners give out to the tenants grain loans to increase consumption up to the minimum p_{a1}. In favorable years the peasants return the debts.

The landowners spend a part of their income for purchase of craft products and to maintain their servants. Let the number of the handicraftsmen be $Y_r(t)$ and they have stocks of grain $X_r(t)$. The minimal general consumption of handicraftsmen is $P_{r0} = p_0 Y_r(t)$. In case $X_r(t) > P_{r0}$ the handicraftsmen have surpluses of grain, and their consumption per capita (p_{ur}) is calculated just as for the peasants. Consumption is $P_r = p_{ur*} Y_r(t)$, and by the next harvest the stocks of the handicraftsmen will be equal $X_r(t) – P_r$. The tax H_r is transformed to a grain equivalent as before. In case the landowners spend for purchase of craft products k_r % of their incomes, the next year the stocks of the handicraftsmen will be $X_r(t + 1) = k_r(lM_a – H_a)/2 – H_r + + X_r(t) – P_r$. The maximal number of the handicraftsmen is determined by $C = P_r/p_0$, and the actual number is determined just as for the peasants. During famines the handicraftsmen receive the grain loans from the landowners and try to increase consumption at least up to the size of a minimum $p_{r1.}$ In favorable years they return the debts together with interest.

The peasants, who have lost the land, engage in craft activity. Some leave for cities, others produce craft products at times not utilized in agriculture. It is useful to consider the peasants and handicraftsmen separately. Assume that four peasants receiving the quarter of their income from craft production produce an equivalent output of three peasants and one craftsman. The handicraftsmen sell goods to buy grain. The peasants live in a natural subsistence economy, and the handicraftsmen sell the goods to landowners. The number of new workplaces of handicraftsmen and servants is limited by the income of landowners received from the tenants of the previous year. Let D_a be areas of the new tenants, and H_{aa} are taxes, paid by them; the income of the landowner from them will be:

$$G = (k_s q\ (l – 1)\ D_a – H_{aa})/2.$$

A share (k_{r1}) of this income is paid to the handicraftsmen, and the number of the new handicraftsmen and servants can be $k_r G/p_0$. Anther part (k_{r2}) of the income of the landowners is stored in terms of grain stocks, and the third part (k_{r3}) is spent for consumption[3]. Let k_{r1} 50%, k_{r3} =25% and the consumption per capita of the landowners is five times more than consumption of the handicraftsmen. Then the number of the landowners will be ten times less than number of handicraftsmen. The maximum number of the landowners is nearly half million.

The state also stores grain. Half of the grain received by the state as the ground tax is accumulated in state barns according to the recommendation of the treatise ‘Kuan-tzu’. The remaining portion was about 2,5 millions tons according to estimates. There were about 150 thousand officials in China. The minimum annual salary of the official was 100 'shih' or about 3 tons of a grain. Probably, for the salary of the officials there required about 1 million tons. The remaining grain was used to supply the army, to pay a contribution to nomads and so on. We suppose, that a share of the public revenues (k_{r4}) was used to buy craft products. The income of the handicraftsmen is increased accordingly.

The values of many of the parameters used are taken from the historical documents. But mathematical models usually contain some arbitrary parameters, which are selected by numerical experiment. In our case the parameter of compensation (n) is the most important one, because it describes the death rate of the population after consumption $p(u)$ falls below the critical level p_0. In traditional models ($n = 1$) the reduction of output below half of p_0 results in a reduction of the population by only 3 %. Such a reduction after a major crisis is obviously implausible. This is also the case for $n = 2$ and $n = 3$. Therefore we shall consider cases $n = 4 – 6$.

The results of simulating the above described model are presented in Fig. 3 (with $n = 6$). The presence of casual fluctuations of productivity causes the variations in the curves. However, the fluctuations of productivity do not influence the population until about the year 100. The calculations show that at that period the peasants had long-term stocks of grain, and the poor harvest did not result in famines. The curve of growth of population is smooth and steady in this period. The calculated population differs from actual population levels estimated on the basis of historical documents only very slightly. The data on the population and sowing areas are taken from (Lee 1921: 436; Krukov, Perelomov, Sofronov and Cheboksarov 1983: 41).

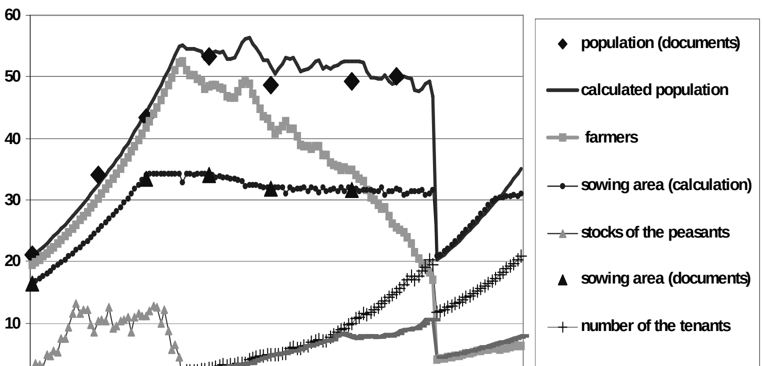

Fig. 3. Economic dynamics in the period Later Han

The general tendencies are also correlated. In the years between 57 and 85 the peasants intensively brought virgin soil into cultivation and possessed large stocks of grain (see Fig. 3). During this period the consumption was large and the population grew quickly. After the year 85 the internal colonization was slowed down owing to gradual exhaustion of reserves of free lands, but the population continued to grow. Hence, per capita consumption began to exceed the currently produced crops and inventory of grain began to decrease.

About the year 102 the stocks were depleted and famines began (Malijvin 1983: 80). The peasants began to sell land owing to famine; this enabled many of them to avoid death through malnutrition, however population decreased a little. The historical records document the first large revolts of peasants. This crisis is of great importance because after it the stability of economic processes was disturbed. Thereafter peasants had no inventories and poor harvests could result in terrible famine and demographic catastrophes (Fig. 4). However, the state tried to maintain stability and during famines officials distributed grain from the state inventories. Catastrophes were thereby avoided for an extended period of time. According to historical evidence after the year 157 the state barns became empty and the distribution of grain stopped (Malijvin 1983: 77).

Then the catastrophe became inevitable. At this time the 'Yellow Turbans' revolt took place and then the long internecine wars began.

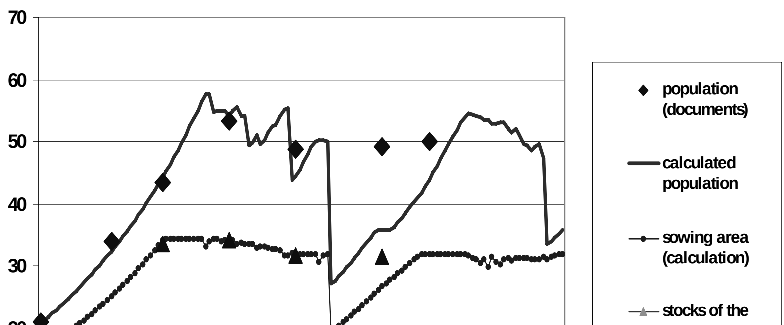

Fig. 4. Economic dynamics in the Later Han period. Variant of calculations in the case of the lack of the state support. Such short demographic cycle took place in China in the Sui epoch.

Thus, the main reason of the catastrophe was the instability of an economy with lack of virgin land and absence of grain inventories. This instability increased when the peasants could freely sell the ground to the landowners. During years 102–160 many peasants were ruined: the poor peasants lived in conditions of incessant famines and sold much of their lands to landowners. The ruined farmers became tenants, handicraftsmen and servants. The number of the farmers was decreasing, but the area of their arable land was decreasing faster. Thus, the disproportion between their number and the area they cultivated grew, bringing about a shortage of cultivated land (Fig. 3). Eventually there remained so little land in their possession, that its sale could no longer rescue the peasants, and a terrible period of famines accompanied by epidemics and revolts began. These catastrophes appear very quickly in the graph. However, in reality the revolts resulted in disintegration of the state and long civil wars. The real losses in lives were more dramatic, than the trends in the graph show[4]. We have demonstrated that the above model can reproduce the salient features of Chinese demographic experience in the 1st and 2nd centuries of our era.

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NOTES

[1] A more detailed exposition of algorithm is available at http://hist1.narod.ru/Science/Cekl/index.htm

[2] But the situation is different in some other cases, for example at the Sung epoch.

[3] The dynamic evolution of the number of landowners is outside of the scope of this study.

[4] The calculations show, that if the peasants had no right to sell the land, their acreage in their possession is more stable. Probably, this was the reason why Oriental monarchies often forbade the peasants to sell their land.

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